ITCS 312: Automata and Formal Languages

Exam 2', Second semester 2007/2008, Form: $\boxed{A}$

Section 1. (1 point each)

Mark the following statements with **True** if they are true and **False** otherwise.

T The language $L = \{a^n b^m : n < m < 7n\}$ can be accepted by an regular grammar.

T The grammar $S \to aSb|ab$ is equivlant to $S \to aAb, A \to aAb|\lambda$.

F The language $L = \{a^n b^m : 2n + 2m \neq 7m\}$ is not regular.

T If we eliminate λ and unit transitions from a grammar then we can use exhaustive search to check membership.

T If L_1 and L_2 are regular then so is $L_1 - L_2{}^*$. $L_1 \cap \overline{L_2^*}$

T Every regular grammar is also a context-free grammar.

T There exists a non-ambiguous grammar for the language $L = \{a^n b^n c^m\} \cup \{a^m b^n c^n\}$.

T Every context-free grammar can be converted to an equivalent grammar which is in Greibach Normal Form.

T The grammar $S \to aS|aaS|A|a, A \to aaa|bbb|\lambda$ is a regular grammar.

T A variable A in a context-free grammar is usefull if $S \Longrightarrow xAy \Longrightarrow w$ for some $w \in L$.

6/10

Section 2. (5 points each)

1. Find a regular grammar for the following language $L = \{a^n b^m a^k : (n + m) \bmod 3 = 0, k \geq 2\}$.

5

$H \to S|A|J$

$S \to aaaS|B$

$B \to bbb|bbbB|D$

$D \to aD|aa$

$A \to aaaA|aC$

$C \to bbbC|bbD$

$J \to aaaJ|aaM$

$M \to bbbM|bD$

mod 3 = 0

mod (n) = 0 + mod(m) = 0

or mod (n) = 1 + mod(m) = 2

or mod (n) = 2 + mod(m) = 1

aabb aaaa

2. Show that the following language $L = \{a^n b^m a^k : k > n + m\}$ is context-free.

5

$S \to aSa \mid bSa \mid A$

$A \to aA \mid a$

aaabbb aaaaaaa

aaabbb aaaaaaaaaa

3. Show that the following language $L = \{a^n b^l : n + l \text{ is a prime number}\}$ is not regular.

3

let $w = a^p b^{p+1} \notin L$? $= xyz$

$a^1 b^2$ $1+2=3$

$|xyz| \leq m$

$|y| \geq 1$

$y = a^k$

$w_i = a^{p+(i-1)k} b^{p+1}$ $1 \leq k \leq m$

$p+(i-1)k + p+1 = p((i-1)k+1)$

$i = \frac{k-2p-1}{k}$ ← alwayze exist because k integer

let $w = a^p b^p = xyz$ $\notin L$

$|xyz| \leq m$

$|y| \geq 1$

$y = a^k$

$1 \leq k \leq m$

$w_i = a^{p(i-1)k} b^p$

$p(i-1)k + p =$

$p((i-1)k+1)$

$\therefore w \notin L$

not regular

4. Show that the following grammar is ambiguous.

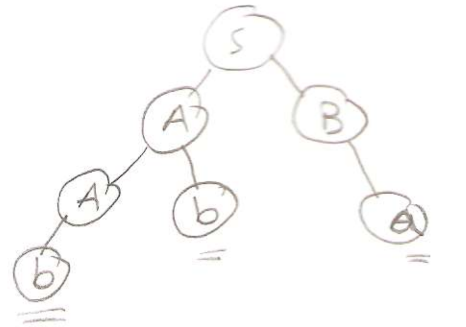
$$
\begin{aligned}
S &\rightarrow AB|bbB \\
A &\rightarrow b|Ab \\
B &\rightarrow a
\end{aligned}
$$

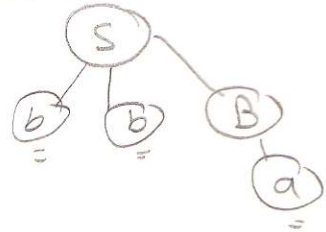
5

$S \Rightarrow AB \Rightarrow AbB \Rightarrow bba$

$S \Rightarrow bbB \Rightarrow bba$

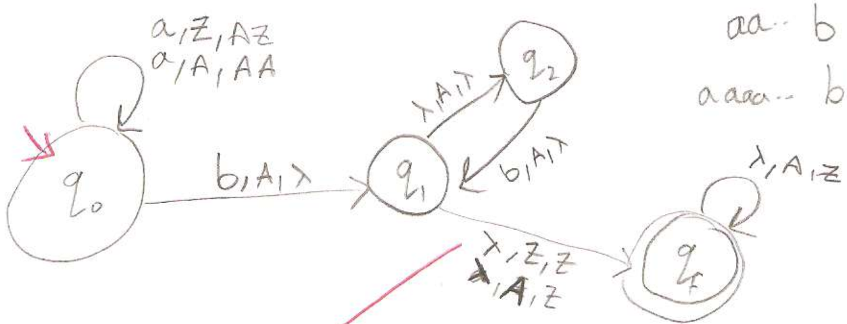
So it is ambiguous





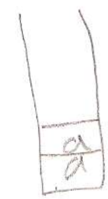
5. Construct an NPDA that accepts the language $L = \{a^n b^m : n \geq 2m\}$.

5



aa.. b

aaaa.. bb



aaaaaaabbb